A Generalization of Friedman's Permanent Income Hypothesis with a Large, Negative Income Shock

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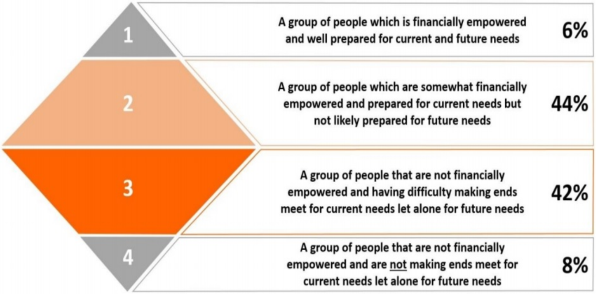
Background

- Unstoppable technological revolution and emerging new forms of work by artificial intelligence and big data analysis
  - Frey and Osborne (2017): 47% of jobs in the U.S. runs the risk of being automated, disrupting labor markets in the long run
- Increased concern about future income uncertainty
- Challenge: How to continue being able to afford when we can currently afford, i.e., how to attain a smooth profile of future consumption?

Background (cont'd)

- Friedman's (1957) permanent income hypothesis (PIH): People should save now to prepare for the aftermath of a permanent decline in income (income shock)
- However, people are not ready to meet their future consumption needs if an income shock occurs



Background (cont'd)

- PIH predictions of Bewley (1977) and Campbell (1987): an income shock hardly affects the precautionary savings of people who are at the higher end of wealth
- However, positive and even high savings rates are very common amongst wealthy people
  - A positive relation between savings rates and income (Dynan *et al.*, 2004), entrepreneurship purposes for entering and expanding business (Quadrini, 1999; Buera, 2009), out-of-pocket medical expenses patterns (De Nardi *et al.*, 2010), the mix of bequests and human capital, entrepreneurship, and medical-expense risk (De Nardi and Fella, 2017)
- **Generalize the PIH with a large, negative income shock (LNIS) and examine its effects on people's precautionary savings behavior**

Contribution

- Precautionary savings rise with wealth, explaining high savings rates of the wealthy
- Substantial amount of extra precautionary savings for consumption smoothing, driven by high-wealth people, could play a role in a decrease of interest rate, which is particularly relevant to today's low interest rates
- Develop an analytically tractable martingale pricing approach in an incomplete market with the LNIS

Literature Review – Optimal Consumption and Investment Framework

- Since Merton (1969, 1971), Farhi and Panageas (2007), Choi *et al.* (2008), and Jang *et al.* (2013) incorporate nontradable income in the Merton framework
- Labor income shocks have been modeled by Brownian motions with the log-normality assumptions (Merton, 1971; Bodie *et al.*, 1992; Heaton and Lucas, 1997; Duffie *et al.*, 1997; Koo, 1998; Cocco *et al.*, 2005; Gomes and Michaelides, 2005; Polkovnichenko, 2007; Benzoni *et al.*, 2007; Wachter and Yogo, 2010; Dybvig and Liu, 2010; Munk and Srensen, 2010; Lynch and Tan, 2011a, 2011b; Calvet and Sodini, 2014; Ahn *et al.*, 2019; Jang *et al.*, 2019)
- Brownian motions cannot explain the effects of low-probability, high-impact events such as forced unemployment and job displacement (Low *et al.*, 2010)

Literature Review – General Equilibrium Analysis

- Standard literature on general equilibrium analysis assumes market completeness (Basak, 1995; Heaton and Lucas, 1996; Basak and Cuoco, 1998; Basak and Shapiro, 2001; Liu *et al.*, 2003; Maenhout, 2004; Gârleanu and Panageas, 2015; Kimball *et al.*, 2018)
- Unhedgeable income shocks (Wang, 2003; Gomes and Michaelides, 2008; Guvenen, 2009; Krueger and Lustig, 2010; Christensen *et al.*, 2012; Dumas and Lyasoff, 2012)
- These models consider only diffusive-type income shocks and cannot account for jump-type income risk
- **Ours considers both diffusive income shocks and LNIS in the equilibrium analysis, and explains today's low-interest-rate environment through the precautionary savings channel**

Literature Review – Market Incompleteness

- Market completeness under no arbitrage: the unique state price density (Ross, 1978; Harrison and Kreps, 1979)
- When markets are incomplete, the number of state price densities is infinite
- The state price density with market incompleteness has been derived explicitly by Kou (2002) and Liu *et al.* (2003)
- However, these models overlook the labor income risk
- Based on Karatzas *et al.* (1991), Liu and Pan (2003) and Branger *et al.* (2017) establish the idealistically completed market

Literature Review – Benchmark Papers

- Wang *et al.* (2016, JET): an incomplete-market consumption-savings model with recursive utility and stochastic income
- Bensoussan *et al.* (2016, OR): a life-cycle model with jump-type forced unemployment risk
- Schmidt (2016, Working Paper): asset pricing of idiosyncratic tail risk with recursive preferences, heterogeneous agents, and incomplete markets
- **None of these, however, investigate precautionary savings implications with the LNIS on interest rates**

Model Setup

- The bond price B and the stock prices S are given by

$$dB(t) = rB(t)dt$$

and

$$dS(t) + D(t)dt = S(t)\{\mu dt + \sigma^\top dZ(t)\},$$

where r is the risk-free interest rate, $D(t) = (d_1, ..., d_N)$ are dividends for N risky stocks, μ is the constant mean vector, σ is the constant nonsingular standard deviation matrix, and $Z(t)$ is the standard Brownian motion process with dimensionality equal to the number of linearly independent returns on stocks.

Model Setup (cont'd)

- Aggregate output process $I(t)$ is assumed to follow a geometric Brownian motion:

$$dI(t) = \mu^I I(t)dt + (\sigma^I)^\top I(t)dZ(t), \quad I(0) = I > 0,$$

  where μ^I and σ^I represent output mean and standard deviation vector, respectively.
- The output is exposed to a large, negative income shock (LNIS), which is distributed according to an exponential distribution (or a Poisson shock) with intensity δ.
- The output is assumed to plummet immediately to $kI(t)$ from $I(t)$ ($k \in (0,1)$) in the aftermath of such a random and jump event.

Model Setup (cont'd)

- We assume that the fraction ξ ($\xi \in (0,1)$) of aggregate output constitutes aggregate earnings $\xi I(t)$.
- The remaining fraction $1-\xi$ of aggregate output is being paid out as dividend as follows:

$$D(t) = (1-\xi)I(t).$$

The Optimal Consumption and Investment Problem

- The optimal consumption and investment problem with the LNIS is given by

$$V(w, I) \equiv \sup_{(c,\pi)} E\Big[\int_0^\infty e^{-(\beta+\delta)t}\Big(\frac{c(t)^{1-\gamma}}{1-\gamma} + \delta K\frac{\{W(t)+k\xi I(t)/\beta_1\}^{1-\gamma}}{1-\gamma}\Big)dt\Big], \quad (1)$$

subject to the following wealth process:

$$dW(t) = \{rW(t) - c(t) + \xi I(t) + \pi(t)^\top(\mu - r\mathbf{1})\}dt + \pi(t)^\top\sigma^\top dZ(t),$$
$$W(0) = w > -\xi I/\beta_1, \quad (2)$$

$$\beta_1 = r - \mu^I + (\sigma^I)^\top\theta, \quad \theta = (\sigma^\top)^{-1}(\mu - r\mathbf{1}),$$
$$W(t) > -\frac{\xi I(t)}{\beta_1}, \quad \text{for all } t \geq 0.$$

- The LNIS makes the maximized expected discounted utility $-\infty$ (or the maximized expected utility $+\infty$).

A New Lower Bound of Wealth

- Impose a catastrophically low time-varying value of wealth, reminiscent of a starvation level below which people cannot sustain themselves financially and consequently, do not invest in the stock market.
- The lower bound of wealth is given by

$$W(t) > -L(t) > -\frac{k\xi I(t)}{\beta_1}, \quad \text{for all } t \geq 0,$$

where $L(t)$ is a given nonnegative time-varying function that makes endogenous investment in the stock market equal to zero.

Optimal Consumption and Investment Strategies

The precautionary savings (PS) driven by the disastrous income shock are given by

$$\begin{aligned}\text{PS} &= \{\text{first term of income risk-driven precautionary savings}\} \\ &\quad + \{\text{second term of income risk-driven precautionary savings}\} \\ &\equiv PS1 + PS2,\end{aligned}$$

where PS1 is given by

$$\frac{2\delta K(\alpha_\delta - 1)\xi I}{||\beta_3||^2(\alpha_\delta - \alpha_\delta^*)(1-\gamma)} z^{-\alpha_\delta} \int_0^z \mu^{\alpha_\delta - 2}\Big(G(\mu) - \frac{1}{\beta_1} + \frac{k}{\beta_1}\Big)^{1-\gamma} d\mu < 0,$$

and PS2 is given by

$$\frac{2\delta K(\alpha_\delta^* - 1)\xi I}{||\beta_3||^2(\alpha_\delta - \alpha_\delta^*)(1-\gamma)} z^{-\alpha_\delta^*} \int_z^{\overline{z}} \mu^{\alpha_\delta^* - 2}\Big(G(\mu) - \frac{1}{\beta_1} + \frac{k}{\beta_1}\Big)^{1-\gamma} d\mu > 0.$$

Optimal Consumption and Investment Strategies (cont'd)

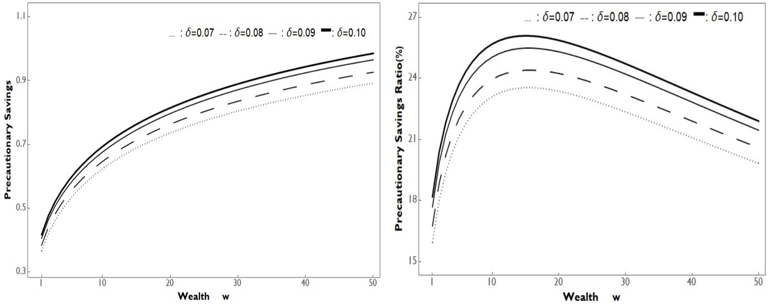
Theorem

The optimal decisions for consumption $c(t)$ and risky stock investment $\pi(t)$ are obtained in closed-form:

$$c(t) = (\hat{A} + \delta)\Big[w + \frac{\xi I}{\beta_1} - \xi I B^*(\overline{z};\delta) z^{-\alpha_\delta^*} - \mathit{PS}\Big],$$

$$\begin{aligned}\pi(t) = &\frac{1}{\gamma}\sigma^{-1}\theta w \\ &+ \frac{1}{\gamma}\sigma^{-1}(\theta - \gamma\sigma^I)\Big[\frac{\xi I}{\beta_1} + (\gamma\alpha_\delta^* - 1)\xi I B^*(\overline{z};\delta) z^{-\alpha_\delta^*} \\ &- \frac{2\gamma}{||\beta_3||^2}\delta K \frac{\left(w + \frac{k\xi I}{\beta_1}\right)^{1-\gamma}}{1-\gamma} \Big/ c(t)^{-\gamma} \\ &+ (\gamma\alpha_\delta - 1)\times \mathit{PS1} + (\gamma\alpha_\delta^* - 1)\times \mathit{PS2}\Big].\end{aligned}$$

Precautionary Savings



- In the presence of the LNIS ($\delta > 0$), the amount of precautionary savings is an increasing and concave function of wealth.
- The possibility of the LNIS increases the percentage precautionary savings as wealth increase, explaining high savings rates of the wealthy rather than negative savings rates predicted by Bewley (1977) and Campbell (1987).

Risky Investments

	Age				
Percentile of Net Worth	35-44	45-54	55-64	65-74	75-80
0-25	6.0	5.7	5.5	1.3	0.5
25-49.9	10.7	10.4	9.7	3.4	2.0
50-74.9	14.8	14.6	14.2	6.2	4.1
75-89.9	17.7	17.6	17.3	9.1	6.7
90-100	20.1	20.0	19.8	12.0	9.4
all	14.8	14.6	14.2	4.1	4.1

- Rules of thumb: As people get older, their risky investment should be geared toward relatively safe assets
- Generates empirically plausible values of 0 to 20% for optimal stock investment
- People's risky investment ratio rises as their wealth increases, consistent with Wachter and Yogo (2010)

Equilibrium Risk-Free Interest Rate

Theorem

The equilibrium risk-free interest rate is derived in closed-form:

$$r = \beta + \gamma\mu^I - \frac{1}{2}\gamma(1+\gamma)(\sigma^I)^2 - (\hat{\delta}(r) - \delta), \tag{3}$$

where μ^I and σ^I represent the expected consumption growth rate and volatility of consumption growth rate, and the constant $\hat{\delta}(r)$ is determined by solving the following non-linear algebraic equation:

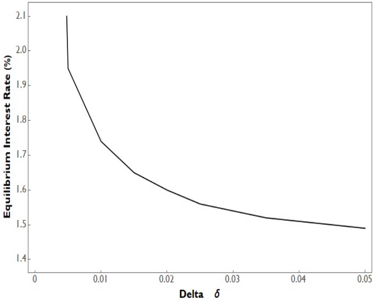
$$\hat{\delta}(r) = \left\{\left(\frac{w}{\xi I} + \frac{1}{\beta_1(\hat{\delta}(r))}\right)\Big/\left(\frac{w}{\xi I} + \frac{k}{\beta_1(\hat{\delta}(r))}\right)\right\}^{\gamma}\{\beta_1(\hat{\delta}(r))\}^{\gamma}\delta K(r)$$

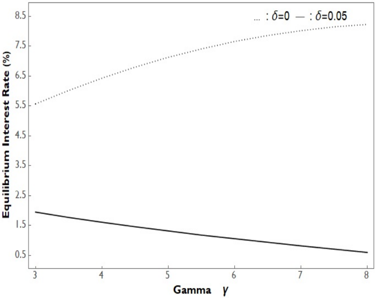
with

$$\beta_1(\hat{\delta}(r)) = \beta + (\gamma - 1)\mu^I - \frac{1}{2}\gamma(\gamma-1)(\sigma^I)^2 - (\hat{\delta}(r) - \delta),$$

$$K(r) = \left\{\frac{\gamma-1}{\gamma}\left(r + \frac{\gamma(\sigma^I)^2}{2}\right) + \frac{\beta}{\gamma}\right\}^{-\gamma}.$$

Equilibrium Risk-Free Interest Rate (cont'd)





- The equilibrium interest rate decreases 64.99% (to 1.96% from 5.57%) as δ increases from 0 to 0.5%
- An increase in risk aversion leads to a decrease in risk-free rate in the presence of the LNIS

Matching Equity Premium and Risk-Free Rate

Estimated consumption and return parameters	1891-1994	1947.2-1996.3
Expected consumption growth rate μ^I	1.74%	1.91%
Consumption volatility σ^I	3.26%	1.08%
Stock Volatility σ	18.53%	15.22%
Risk-free rate r	1.96%	0.79%
Equity premium $\mu - r$	6.26%	7.85%

Data

Required preference parameter	1891-1994	1947.2-1996.3
Risk aversion (γ)	7.5	10
Risk-free rate (r)	1.64%	1.35%
Equity premium $(\mu - r)$	4.53%	1.64%

Ours: $\delta = 0.005$ and $\beta = 0.03$

- The presence of the LNIS improves the model's ability to match asset prices
- A moderate risk aversion coefficient of 7.5 explains the equity premium of 4.53% and the risk-free rate of 1.64%

Conclusion

- Generalize Friedman's permanent income hypothesis (PIH) by considering the possibility of a large, negative income shock (LNIS)
- Find an increasing and concave trend of the savings with respect to wealth, thereby explaining why high-wealth people save more than low-wealth people
- The equilibrium interest rate falls dramatically even when the chance of the LNIS is small; this result partly explains today's low interest rates

Thank you very much!